MATHEMATICS (SEMESTER - 2)

CS/BCA/SEM-2/BM-201/08

1. .....
Signature of Invigilator

2. .....
Signature of the Officer-in-Charge

Reg. No.

Roll No. of the Candidate

CS/BCA/SEM-2/BM-201/08

ENGINEERING & MANAGEMENT EXAMINATIONS, JUNE – 2008

MATHEMATICS (SEMESTER - 2)

Time : 3 Hours] [Full Marks : 70

INSTRUCTIONS TO THE CANDIDATES :

1. **This Booklet is a Question-cum-Answer Booklet.** The Booklet consists of **32 pages**. **First page** of the Booklet shows Instructions to the Candidates. The **questions** of this concerned subject commence from **Page No. 3**.
2. a) In **Group – A**, Questions are of Multiple Choice type. You have to write the correct choice in the box provided **against each question**.
   b) For **Groups – B** & **C** you have to answer the questions in the space provided marked 'Answer Sheet'. Questions of **Group – B** are Short answer type. Questions of **Group – C** are Long answer type. Write on both sides of the paper.
3. Fill in your Roll No. in the box provided as in your Admit Card before answering the questions.
4. Read the instructions given inside carefully before answering.
5. You should not forget to write the corresponding question numbers while answering.
6. Do not write your name or put any special mark in the booklet that may disclose your identity, which will render you liable to disqualification. Any candidate found copying will be subject to Disciplinary Action under the relevant rules.
7. Use of Mobile Phone, Calculator or Log table is totally prohibited in the examination hall.
8. You should return the booklet to the invigilator at the end of the examination and should not take any page of this booklet with you outside the examination hall, which will lead to disqualification.
9. Rough work, if necessary is to be done in this booklet only and cross it through.

No additional sheets are to be used and no loose paper will be provided

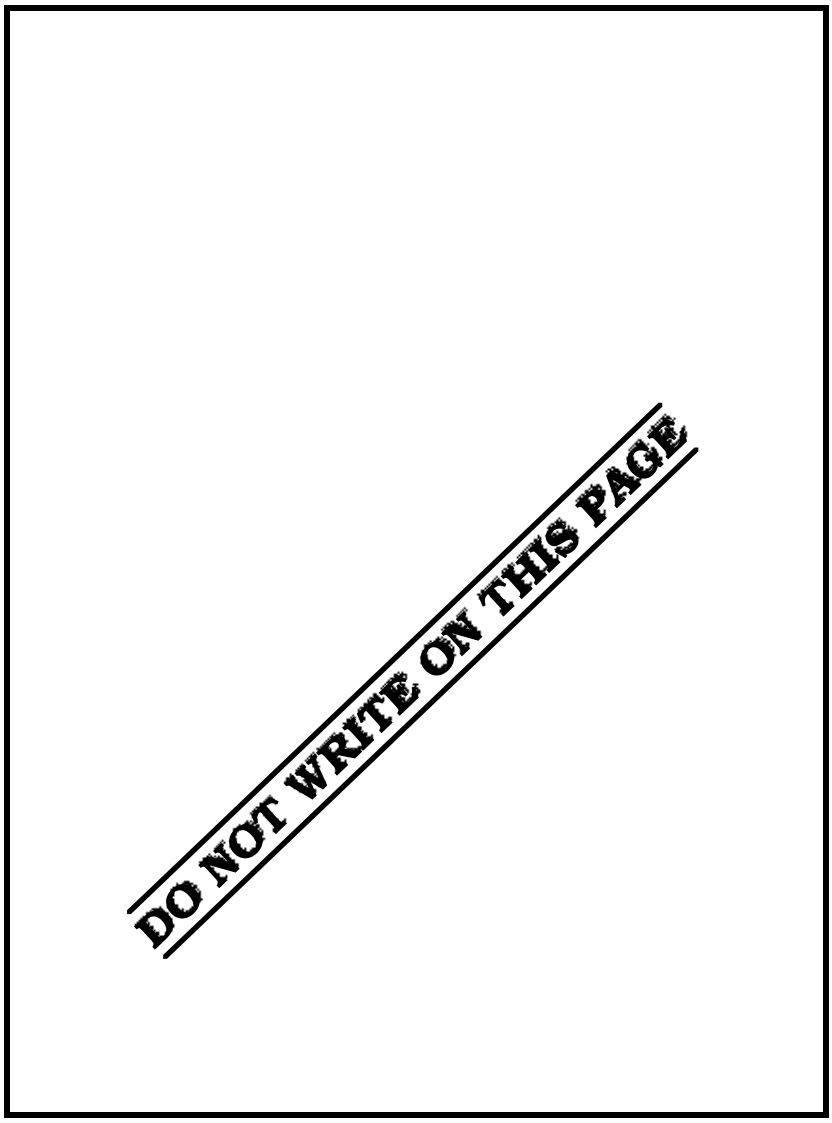
---

FOR OFFICE USE / EVALUATION ONLY

Marks Obtained

	Group – A										Group – B			Group – C				
Question Number																	***Total Marks***	***Examiner's Signature***
Marks Obtained																		

.....
Head-Examiner/Co-Ordinator/Scrutineer


DO NOT WRITE ON THIS PAGE

ENGINEERING & MANAGEMENT EXAMINATIONS, JUNE – 2008

MATHEMATICS

SEMESTER – 2

Time : 3 Hours] [Full Marks : 70

GROUP – A

(Multiple Choice Type Questions)

1. Choose the correct alternatives for any *ten* of the following : $10 \times 1 = 10$

i) The order and degree of the differential equation

$$\left(1 + \frac{d^2y}{dx^2}\right)^{3/2} = a\frac{d^2y}{dx^2}$$ are

a) 2, 1 b) 2, 2

c) 2, 3 d) none of these. ☐

ii) The particular integral of $\frac{d^3y}{dx^3} - 4\frac{dy}{dx} + 9y = e^{2x}$ is

a) $\frac{e^{2x}}{9}$ b) $\frac{e^{2x}}{3}$

c) $\frac{e^{2x}}{6}$ d) none of these. ☐

iii) Auxiliary equation of the differential equation $\frac{d^2y}{dx^2} + 4y = \sin x$ is

a) $y = \cos 2x + \sin 2x$ b) $y = c_1 \cos 2x + c_2 \sin 2x$

c) $y = c_1 \cos x + \sin 2x$ d) none of these. ☐

iv) Integrating factor of $x\,dy = y\,dx$ is

a) $\frac{1}{x^2}$ b) $\frac{1}{x}$

c) $\frac{y}{x}$ d) $\frac{1}{y^2}$. ☐

v) A bounded sequence is

a) convergent b) divergent

c) oscillatory d) none of these. ☐

vi) A monotone increasing sequence is

a) bounded above b) bounded below

c) convergent d) divergent. ☐

vii) If $\lim_{n \to \infty} a_n = 0$, then the series $\sum_n (-1)^n a_n$ is

a) convergent b) divergent

c) none of these. ☐

viii) The infinite series $\frac{n}{(n+1)^2(n+2)}$ is

a) convergent b) divergent

c) oscillatory d) none of these. ☐

ix) The dimension of the subspace $\{ (x, 0, y, 0) ; x, y \text{ are real} \}$ is

a) 1 b) 2

c) 3 d) 4. ☐

x) If $\sum_n |a_n|$ is convergent then $\sum_n a_n$ is

a) convergent b) divergent

c) oscillatory d) none of these. ☐

xi) Sum (union) of two subspaces of a vector space is a

a) subspace of the vector space b) not a subspace of the vector space

c) none of these. ☐

xii) $T : \mathbb{R}^2 \to \mathbb{R}$ is defined by $T(x_1, x_2) = x_1, x_2$. Then kernel of T is

a) $\{(x_1 - x_1) : x_1$ is real $\}$ b) $\{(0, 0), (1, -1)\}$

c) $\{(0, 0)\}$ d) $\{(2, -2)\}$. ☐

xiii) The series $\sum_{n=1}^{\infty} \frac{1}{n^p}$ is convergent if

a) $P \geq 1$ b) $P > 1$

c) $P < 1$ d) $P \leq 1$. ☐

GROUP – B

(Short Answer Type Questions)

Answer any *three* of the following. $3 \times 5 = 15$

2. Test the convergence of the series :

$$x + \frac{2^2 x^2}{2!} + \frac{3^3 x^3}{3!} + \frac{4^4 x^4}{4!} + \ldots\ldots\ldots\ldots\ldots, \; x > 0.$$

3. Examine conditional convergenee of the series.

$$1 - \frac{1}{3} + \frac{1}{5} - \frac{1}{7} + \ldots\ldots\ldots\ldots\ldots$$

4. Show that the differential equation of all circles touching the x-axis at origin is

$$(x^2 - y^2)\frac{dy}{dx} = 2xy.$$

5. Let V = set of all second order sqaure matrix. $T : V \to V$ is defined by

$T(X) = AX - XA$, where $A = \begin{bmatrix} 1 & 2 \\ 0 & 3 \end{bmatrix}$. Find a basis of Ker (T) and hence nullity.

6. Define a monotonic sequence. When is a monotone sequence convergent ? Is the sequence $\left\{\frac{3n + 1}{n + 2}\right\}_n$ convergent ?

GROUP – C

(Long Answer Type Questions)

Answer any *three* questions. $3 \times 15 = 45$

7. a) Examine if the set S is a subspace of R^3 where,

$S = \{ (x, y, z) \varepsilon R^3 : x + 2y - z = 0, 2x - y + z = 0 \}$. If S be a subspace determine its dimension.

b) If $\{ \alpha, \beta, \gamma \}$ is a basis of a real vector space V, show that $\{ \alpha + \beta, \beta + \gamma, \gamma + \alpha \}$ is also a basis of V.

c) Show that the vectors $(1, -2, 3)$, $(2, 3, 1)$ and $(-1, 3, 2)$ form a basis of R^3. Determine co-ordinates of $(1, 0, 0)$ relative to this basis. $5 + 5 + 5$

8. a) A linear mapping $T : R^3 \to R^2$ is defined by

$T (3x - 2y + z, x - 3y - 2z), (x, y, z) \varepsilon R^3$.

Find the matrix of T relative to the ordered bases $(0, 1, 0)$, $(1, 0, 0)$, $(0, 0, 1)$ of R^3 and $(0, 1)$, $(1, 0)$ of R^2.

b) Determine the linear mapping $T : R^3 \to R^3$ that maps the basis vectors $(1, 0, 0)$, $(0, 1, 0)$, $(0, 0, 1)$ to the vectors $(-1, 2, 1)$, $(1, 1, 2)$, $(2, 1, 1)$ respectively. Find Ker T and verify that dim Ker T + dim Im T = 3.

c) If $V(F)$ is the vecror spaces of all $2 \infty 2$ matrices then exibit a basis for $V(F)$ and also find the dimension. $5 + 5 + 5$

9. Test the convergence of any *three* of the following series. 3×5

a) $1 + \frac{2}{1!} + \frac{2^2}{2!} + \frac{2^3}{3!} + \ldots\ldots\ldots\ldots$

b) $\frac{1}{2} + \frac{1}{3} + \frac{1}{2^2} + \frac{1}{3^2} + \frac{1}{2^3} + \frac{1}{3^3} + \ldots\ldots\ldots\ldots$

c) $\sum_{n=1}^{\infty} \frac{n! \, 2^n}{n^n}$

d) $\frac{1}{1 + a^2} - \frac{1}{2 + a^2} + \frac{1}{3 + a^2} - \ldots\ldots\ldots\ldots$

10. Solve any *three* of the following : 3 × 5

a) $\frac{dy}{dx} + \frac{1}{x} \sin 2y = x^3 \cos^2 y$

b) $y = px + p^n ; p = \frac{dy}{dx}$

c) Solve $\frac{d^2y}{dx^2} + a^2y = \sec ax$

d) $x^2 \frac{d^2y}{dx^2} + 4x \frac{dy}{dx} + 2y = \log x.$

11. a) What do you mean by convergence of a sequence ? Examine convergence of a sequence $\{x_n\}$, where $x_n = 1 + \frac{1}{2} + \frac{1}{3} + \ldots\ldots\ldots + \frac{1}{n}$.

b) If $x_1 = \sqrt{2}$, $x_{n+1} = \sqrt{2xn}$. Show that the sequence $\{x_n\}$ is monotonically increasing and bounded. Hence show that $\lim_{n \to \infty} x_n = 2$.

c) Prove that every convergent sequence is bounded. Is the converse true ? Justify your answer. 5 + 5 + 5

END